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Identifying stages of pattern recognition. Evidence for a hierarchical stage model

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Mathematics can be understood as the science of studying patterns and structures. The ability to recognize patterns is a fundamental prerequisite for this. In addition, there is an empirical link between the ability to recognize patterns and the development of mathematical understanding. It is assumed that pattern recognition is an active design process that is highly individualized. However, studies have shown that success in recognizing patterns depends on various characteristics of the presented sequences, such as whether they are repeating or increasing, and on the ability to identify these characteristics. Given the wide variety of patterns observed across the different studies, it is difficult to make general statements about individual developmental stages. Thus, this study aims to investigate whether distinct developmental profiles of pattern recognition can be identified at the individual level based on systematically varied sequences of repeating and growing patterns. Our findings confirm a three-stage model that begins with the concept of the mere regularity of patterns. Next is the ability to consider a basic unit when continuing repeating pattern sequences. By the end, children can identify growing patterns. Furthermore, we could show that students' membership in the identified developmental profiles is associated with grade level, other cognitive covariates, and gender.

KEYWORDS
cognitive development, elementary school, gender, growing and repeating structure, mental rotation, pattern recognition, primary school

1 Introduction

Sawyer (1955) conceptualized mathematics as the discipline concerned with the classification and study of all conceivable patterns. This perspective, which frames mathematics as the science of patterns and structures, has been widely recognized and developed in the literature (Devlin, 1996; Steen, 1988; Wittmann, 2021). Within this framework, mathematical activity is understood to be fundamentally concerned with patterns. Central to this is the ability to identify patterns and structures, a process that relies on a complex interplay between perception and interpretation (Lüken, 2012; Reuter and Bruns, 2022). In line with Battista and Clements (1996), we argue that individuals not only perceive structures in objects, but also construct these structures through mental actions related to the objects. The key challenge is to abstract from individual objects and identify generalizable relationships between them (Steinbring, 2005).

Mulligan and Mitchelmore (2009) introduced the concept of "Awareness of Mathematical Pattern and Structure" (AMPS), emphasizing that this awareness varies widely across individuals and is strongly related to other mathematical competencies. This notion of varying degrees of awareness is supported by studies such as those of

Wijns et al. (2021) and Lüken (2023), which document considerable individual differences in children's spontaneous pattern recognition, even in contexts not explicitly designed to elicit such behavior. Numerous studies have demonstrated a strong predictive relationship between pattern and structure recognition and overall mathematical achievement (Junker et al., 2024; Mulligan and Mitchelmore, 2009; Nguyen et al., 2016; Pasnak et al., 2016; Rittle-Johnson et al., 2017, 2019; Wijns et al., 2021b). Furthermore, developing students' skills in this area has been shown to improve mathematical achievement (Hendricks et al., 2006; Kidd et al., 2013; Lüken and Kampmann, 2018; Mulligan et al., 2020; Pasnak et al., 2015).

Instruction that emphasizes patterns and structures can move learners beyond procedural knowledge and recursive pattern recognition to deeper conceptual understanding, including an awareness of pattern units and their repeatings (Lüken and Sauzet, 2021). Research highlights the importance of communicative processes, i.e., discussing observed structures with students, in supporting the development of structural insight (Björklund and Pramling, 2014; Flynn et al., 2020). The pedagogical importance of engaging with patterns and structures is explicitly reflected in the mathematics curricula of many countries (e.g., Australia: ACARA, 2021; Department of Basic Education, Republic of South Africa, 2012; USA: National Council of Teachers of Mathematics, 2000; Germany: Ständige Konferenz der Kultusminister, 2022).

Finally, empirical evidence suggests that the ability to recognize patterns tends to improve with age (Burgoyne et al., 2017; Clarke et al., 2008; Kyriakides and Gagatsis, 2003; Mulligan et al., 2020; Reuter and Reinhold, 2026). Different types of patterns as well as different levels of difficulty within the patterns play a role here. For example, studies show that, especially for younger children, repeating pattern sequences are challenging to varying degrees depending on the length of the basic unit and the number of different elements that the basic unit contains, and increases as more aspects must be considered (Clements and Sarama, 2009; Lüken, 2012b; Lüken and Sauzet, 2021), while for primary school children, the length of the basic unit seems to be less challenging than the number of different elements that the basic unit contains (Reuter and Reinhold, 2026). With regard to the different pattern types, repeating pattern sequences (e.g., ababab) seem to be easier than pattern sequences with a growing structure (e.g., abaabbaaabbb) (Kyriakides and Gagatsis, 2003; Reuter and Reinhold, 2026; Warren, 2005; Wijns et al., 2021a; Wijns et al., 2021b). However, it is still unclear to what extent overarching individual developmental trajectories in pattern recognition can be identified depending on the difficulty characteristics present in the specific pattern and the two pattern types with repeating and growing structure. Thus, this appears relevant for adequate teaching support to provide children with targeted support in the development of their pattern and structure recognition skills—and to further support mathematical learning. The present study therefore investigated the question of what developmental trajectories can be expected in primary school children with regard to pattern recognition in different pattern types and whether and to what extent gender, age and the ability for mental rotation are related to this.

1.1 Repeating and growing patterns

Although there is no universally accepted definition of the term *'pattern'* within the academic literature, a shared conceptual core can be identified, i.e., that patterns are characterized by some form of regularity (Lüken, 2012a; Mulligan and Mitchelmore, 2009; Sawyer, 1955). Mulligan and Mitchelmore (2009) define a pattern „as any predictable regularity, usually involving numerical, spatial or logical relationships" (p. 34). In order to identify such regularity, it is necessary to identify the underlying structure—that is, the organizational framework of the pattern itself.

Papic and colleagues (2011) categorize patterns into three different types: repeating patterns, growing patterns, and spatial structure patterns. Repeating patterns (e.g., ABABAB) are defined by the recurrence of certain elements and require the identification of a basic unit (e.g., AB or BA) that is repeated. In contrast, growing patterns (e.g., ABAABBAAABBB) are characterized by a progressive increase in elements following a specific rule, requiring recognition of how elements evolve over time to form a coherent structure of change. The third category, spatial structure patterns, involves identifying relationships between different attributes of geometric figures, such as shape, size, or orientation.

The distinctions between these pattern types lie primarily in the nature of the relationships between their constituent elements, therefore pertain to the structure (Lüken and Sauzet, 2021). Whereas both repeating and growing patterns typically occur in linear sequences that can be extended indefinitely, spatial structure patterns are generally restricted to two-dimensional geometric configurations (Wijns et al., 2021b). In both pattern types, the underlying structure depends on the number of elements and the length of a basic unit, with growing pattern sequences supplemented by the question of which and how many of the elements are affected by the growing structure (Baumanns et al., 2024; Reuter and Reinhold, 2026). Analyses of individual approaches to pattern sequences reveal that patterns initially designed to exhibit a growing structure can be reinterpreted or transformed into repeating patterns, e.g., by continuing the initial segment in a repeating manner. Such continuations may still align with the overarching principle of regularity. This observation suggests that a strictly normative evaluation based on a binary correct-or-incorrect framework is inadequate for assessing pattern-related thinking, as the boundaries between different pattern types may be fluid and context-dependent. Accordingly, in the present study, first at all a pattern is defined as any sequence that displays regularity, regardless of whether the continuation aligns with the originally intended structure—as long as the basic unit appears at least twice. This is a basic criterion for correctness, but it aligns with the generally accepted view of patterns, which assumes regularity is a fundamental requirement (see above). Next, the extent to which the originally intended structure is evident in the continuation is assessed in more detail.

1.2 Importance of pattern and structure recognition for mathematic learning

The ability to recognize patterns and structures plays a central role in the development of mathematical understanding. A robust body of empirical research supports the strong link between pattern and structure recognition and overall mathematical achievement (Lüken, 2012b; Mulligan and Mitchelmore, 2009; Nguyen et al., 2016; Rittle-Johnson et al., 2019; Schmerold et al., 2017; Zippert et al., 2019, 2020). Furthermore, this ability has been shown to significantly influence the acquisition of specific mathematical skills, including counting, early arithmetic, and basic concepts of multiplication and measurement (Lüken et al., 2014; Mulligan and Mitchelmore, 1997, 2009; Nunes et al., 2008). Early exposure to pattern sequences has also been linked to the development of functional reasoning, which serves as a foundation for algebraic reasoning (Carraher and Schliemann, 2007; Clements et al., 2007; Threlfall, 1999). In other domains of mathematical learning, pattern recognition is considered fundamental to both computational and functional thinking (Acosta et al., 2024). Because conceptual understanding in these domains relies heavily on structural awareness, the recognition and application of patterns and structures is a fundamental prerequisite (Mulligan and Mitchelmore, 2009).

Furthermore, children's ability to recognize repeating and growing patterns at an early age serves as a reliable predictor of later success in school mathematics (Burgoyne et al., 2019; Fyfe et al., 2017; Kidd et al., 2014; Lüken, 2012; Rittle-Johnson et al., 2017; Wijns et al., 2021b; Zippert et al., 2019, 2020). From a developmental perspective, targeted support for pattern and structure recognition has been shown to foster improvements in both general mathematical competence and arithmetic skills (Hendricks et al., 2006; Kampmann, 2015; Kidd et al., 2013; Lüken and Kampmann, 2018), as well as the ability to identify patterns (Clements et al., 2007; Kidd et al., 2013; Klein and Starkey, 2003; Strauss et al., 2020; Wijns et al., 2021a; Zippert et al., 2021).

1.3 Handling pattern sequences: development of pattern recognition

In addition to examining the relationship between the ability to recognize patterns and structures and mathematical proficiency, empirical studies have also examined how preschool and school-age children engage with pattern sequences and the developmental progress observed in this context. These investigations focus on children's engagement with pattern sequence activities and their success rates in completing related tasks.

To describe how children interact with pattern sequences, research has focused on different pattern-related activities or strategies, such as reproducing, extending, and extrapolating patterns (Burgoyne et al., 2017; Clarke et al., 2008; Larkin et al., 2024; Lüken and Sauzet, 2021; Papic and Mulligan, 2007; Steinweg, 2001). Kyriakides and Gagatsis (2003) proposed a stage model for pattern sequence activities, suggesting that children first excel at reproducing patterns, followed by the ability to extend them. At the third stage, children can verbally articulate pattern formation rules. Similar stage models are presented by other research groups (Clarke et al., 2006; Papic et al., 2011; Rittle-Johnson et al., 2015; Starkey et al., 2004). In addition, targeted linguistic support focusing on abstract formulations has been shown to increase children's attention to structural elements and improve their success in pattern sequencing tasks (Björklund and Pramling, 2014; Flynn et al., 2020).

In addition to activities involving pattern continuation and verbalization, Steinweg (2001) identified different levels of understanding that reflect increasing depth in pattern recognition. At the initial level, implicit recognition of patterns is sufficient to continue the sequence. In the next stage, this recognition extends to the ability to describe the pattern. Only at the third level can children justify the connections they observe. While children often remain at a concrete level of understanding, focusing on specific pattern elements when describing and generalizing, moving to an abstract verbalization that is detached from these concrete elements is a significant challenge (Steinweg, 2001).

Mulligan et al. (2020) outlined five stages of individual differences in pattern and structure recognition, ranging from limited and fragmented perception of pattern features to "accurate, efficient, and generalized use of the underlying structure" (Mulligan et al., 2020, p. 668). All studies show that the success rate in processing pattern tasks increases with age (Burgoyne et al., 2017; Clarke et al., 2008; Kyriakides and Gagatsis, 2003; Mulligan et al., 2020; Reuter and Reinhold, 2026).

It can be seen that for younger children the difficulty of patterns depends on complexity-inducing characteristics, such as, the length of the basic unit, and the number of different elements contained in the basic unit (Clements and Sarama, 2009; Lüken and Sauzet, 2021). For primary school children, however, the length of the basic unit no longer seems to be a crucial factor—here it is the number of elements that still proves challenging: the more different elements the basic unit contains, the more difficult it is to continue (Reuter and Reinhold, 2026). Few of the available studies systematically distinguished between pattern sequences ending with an incomplete or complete base unit when selecting the pattern sequences presented. Studies that did take this into account were able to show that the continuation of pattern sequences presented with an incomplete base unit allows conclusions to be drawn about the recognition of the underlying structure in the form of the base unit (Reuter and Reinhold, 2026; Tsamir et al., 2017). It has been shown that it is more difficult for children to continue such pattern sequences while taking the basic unit into account (Reuter and Reinhold, 2026; Tsamir et al., 2017).

Furthermore, growing pattern sequences appear to present greater challenges for preschool and primary school children compared to repeating patterns. This difficulty can be attributed to the more complex underlying structures of growing patterns, which are not immediately apparent visually and must instead be conceptualized mentally (Pasnak et al., 2016; Reuter and Reinhold, 2026; Warren, 2005; Wijns et al., 2019, 2021b). In summary, for primary school children, the difficulty of repeating pattern sequences increases with the number of symbols in the basic unit. However, growing pattern sequences are not affected by this (Reuter and Reinhold, 2026).

A promising approach to understanding and supporting these competencies is the concept of learning trajectories. A learning trajectory consists of a specific mathematical goal, a developmental progression (a path of learning along which children develop more sophisticated levels of thinking), and a set of instructional tasks matched to each level (Clements and

Sarama 2009). These learning trajectories include various pattern activities at different levels, demonstrating varying degrees of success for example with repeating pattern sequences with different difficulties. Regarding the pattern activity "extending," Reuter and Reinhold (2026) have shown that for primary school children no additional difficulty-generating features can be identified within repeating and growing patterns in terms of their underlying structure. Recent interdisciplinary research suggests that these trajectories might not be limited to the mathematical domain. The 'Sounds of Intent' (SoI) framework, for instance, proposes that the acquisition of pattern-processing capacity follows an equivalent trajectory in both the auditory (musical) and visuo-spatial domains (McCarthy et al., 2025; Ockelford et al. 2025). According to Zygonic Theory, which underpins the SoI framework, this progression is driven by the increasing complexity of perceived relationships, moving from individual 'events' (identity of features) to 'groups' (identity of relationships).

1.4 Spatial ability, gender and pattern recognition

In addition to the ability to recognize patterns and structures, spatial ability has also been shown to be (predictively) related to mathematical performance. This means that these two areas are of great importance for early (mathematical) development, but in many studies they have been considered independently of each other. It's not unreasonable to think that the two abilities may be beneficial to each other. For example, it is conceivable that spatial skills may be advantageous in recognizing the structural properties of a pattern and in mentally relating them to each other. The studies by Collins and Laskin (2015) and Rittel-Johnson and colleagues (2019) support this notion: both groups of researchers were able to demonstrate moderate correlations between visuospatial skills and pattern recognition skills, suggesting that visuospatial skills are used during pattern recognition (Rittle-Johnson et al., 2019).

Up to our knowledge potential gender differences in dealing with patterns are not frequently reported in previous studies, only Lüken and Sauzet (2021) found significant gender differences in favor of girls in their study, both with regard to the development of strategies for dealing with repeating pattern sequences and the success rate. These differences are not related to the children's mathematical performance, which was also surveyed and in which no gender differences were found. This suggests that these uniquely observed gender differences should be investigated in further studies. In contrast to this perspective, which has not been widely reported, there is a great deal of evidence for gender differences in the context of spatial ability and gender, especially in the subfield of mental rotation ability. Here, studies show differences in favor of male subjects from the age of 10 years; for younger children, the results are (so far) not clearly investigated (Bott et al., 2023; Grüßing, 2012; Hoyek et al., 2012; Levine et al., 2016; Linn and Petersen, 1985; McGee, 1979; Peters et al., 1995, 2006; Voyer et al., 1995). In this context, a comparative perspective of the two domains of patterning and mental rotation, taking into account a possible gender effect, seems to be of interest.

1.5 The present study

Because the ability to identify patterns and structures can influence mathematical engagement, the importance of pattern and structure recognition for mathematical development is widely recognized (Björklund et al., 2020; Tsamir et al., 2017). Available research indicates that pattern recognition ability increases with age and that growing pattern sequences are more challenging than repeating pattern sequences. Other studies describe difficulty-generating features for repeating pattern sequences in younger children, as the pattern complexity depends on the length of the basic unit and the number of different elements contained in the basic unit (Clements and Sarama, 2009; Lüken and Sauzet, 2021), whereas in primary school age only the number of different elements contained in the basic unit seems to be relevant here (Reuter and Reinhold, 2026). Furthermore, they have shown that for items with a growing structure, it is irrelevant for the recognition of the growing structure whether the presented pattern sequence represents a complete or an incomplete structure: either children recognize the growing structure in both cases or they do not recognize it in either case. This is in contrast to repeating pattern sequences, where it has been shown that the success rate for incomplete stimuli is lower when the analogous consideration of the basic unit in the solutions is taken into account. Most existing studies examine age-related differences at the group level, focusing on average solution rates or item difficulty. Taken together, prior research provides strong evidence that pattern recognition develops with age and that repeating and growing patterns differ in difficulty. However, this evidence is primarily based on group-level comparisons of mean performance and item difficulty. It therefore remains unclear whether children's pattern-recognition abilities are organized into qualitatively distinct individual profiles that reflect hierarchical developmental stages. This is the central problem addressed in the present study. Building on previous findings regarding the development and complexity of pattern recognition abilities, the present study aims to investigate whether distinct individual profiles of pattern recognition can be identified. Therefore, the first research question addresses whether such profiles can be empirically identified.

RQ1: Can distinct individual profiles of pattern recognition abilities be identified? *Individual stage hypothesis: Pattern recognition abilities can be described by a hierarchical model consisting of three individual levels of increasing structural understanding: (1) the ability to produce a regular pattern continuation, (2) the recognition of the basic unit in repeating patterns, and (3) the recognition of growing structures. The model assumes that mastery of higher levels builds upon mastery of the preceding levels. We hypothesize that this theoretically derived stage model is reflected in the empirical data. Specifically, children's performance profiles are expected to correspond to the three proposed levels of pattern recognition, both when individuals are classified according to the theoretical model and when patterns in the data are identified using data-driven procedures.*

In addition to describing such profiles, it is important to understand how they relate to broader developmental factors. Previous research suggests that pattern recognition abilities increase with age and are closely related to mathematical competence. Furthermore, visuospatial abilities may play a role in recognizing structural regularities in patterns. To explore these relationships, we additionally examined whether profile membership is associated with relevant learner characteristics.

RQ2: How is membership in the identified pattern recognition profiles related to grade level, mathematical ability, and visuospatial skills (mental rotation)? *Developmental and cognitive predictors hypothesis: Higher developmental levels of pattern recognition are expected to be associated with higher grade levels and stronger visuospatial abilities, as measured by mental rotation.*

2 Method

2.1 Sample

The sample comprised $N = 387$ primary school students (200 boys, 187 girls) from Grades 1–4. Specifically, 159 students were enrolled in Grade 1 ($M_{age} = 7.01$), 64 in Grade 2 ($M_{age} = 7.98$), 94 in Grade 3 ($M_{age} = 8.97$), and 70 in Grade 4 ($M_{age} = 9.87$). Age was assessed via student self-report in years at the time of testing. The sample was recruited using a convenience sampling procedure involving primary schools in the region. Participating classes were tested as intact classroom groups during regular mathematics lessons. Participation required school-level agreement, written parental consent, and children's verbal assent. Accordingly, the sample reflects the participating school contexts and was not intended to constitute a random or representative sample of the broader population of primary school students.

Part of the data used in the present study has been previously analyzed in Reuter and Reinhold (2026), focusing on different research questions. The current study extends this dataset by incorporating additional measures and analyses.

Data collection was conducted with prior approval from the responsible school authority (Regional Council of Freiburg, 7–6499.2). Participation required written informed consent from parents as well as verbal assent from the children.

2.2 Material and measures

2.2.1 Pattern recognition

Pattern recognition was assessed using an instrument introduced in Reuter and Reinhold (2026). The present study uses the same item set and coding approach but focuses on different research questions. For full details on item construction, coding manuals, and exemplary student responses, we refer to the original publication.

The instrument consists of 38 pattern sequence items that systematically vary with respect to structural properties. Two main types of patterns are included: repeating patterns and growing patterns. Repeating patterns are based on established sequence constructions (Vitz and Todd, 1967, 1969) and vary, for example, in the length of the underlying basic unit, the number and frequency of symbols, and whether the stimulus presents a complete or incomplete basic unit. Growing patterns were constructed by systematically varying the number of elements and the components affected by change, again including both complete and incomplete stimulus presentations. Across all items, a small set of visually simple and clearly distinguishable symbols was used to avoid confounding effects of semantic associations.

Students were asked to continue each pattern sequence. Figure 1a shows a sample item from the pattern recognition instrument as it was presented to the students. Responses were analyzed using a theory-driven coding scheme focusing on how students applied regularities and structural features in their continuations. Coding was conducted at the level of individual item responses and followed a two-step logic.

In a first step, all responses were coded with respect to whether they constituted an errorless continuation that fulfilled the requirement of regularity, irrespective of whether the underlying structure was fully captured. This category ("errorless individual pattern") thus represents a low-threshold correctness criterion and includes, for example, correct continuations by repeating the visible sequence, by taking up the basic unit, or by extending an underlying growth pattern *(i.e., continuation is errorless and fulfils the requirement for regularity)*.

In a second step, more fine-grained structural aspects were analyzed for theoretically selected subsets of items. For repeating patterns with incomplete presentation of the basic unit, coding focused on whether students correctly reconstructed and applied the underlying unit when continuing the sequence. This resulted in the category incomplete pattern recognition, capturing whether the continuation follows the pattern structure rather than merely extending the visible sequence *(i.e., the continuation follows immediately after the end of the pattern start while taking the basic unit into account)*.

For growing patterns, coding focused on whether students identified and continued the underlying growth rule, resulting in the category growing structure recognition *(i.e., the continuation reflects the systematic growth structure, such as adding one element at each step, rather than reverting to repetition or mixed strategies)*. To avoid confounding structural demands, this analysis was restricted to items with complete presentation of the growth pattern.

These coding procedures resulted in three scales: (1) errorless individual pattern ($k = 38$), (2) incomplete pattern recognition ($k = 11$), and (3) growing structure recognition ($k = 7$). Scale scores were computed as proportions of correctly coded responses. Internal consistencies ranged from acceptable to excellent ($\alpha = .72$,.97, and.99, respectively). Descriptive statistics indicated high performance for errorless pattern continuation ($M = 0.91$, $SD = 0.10$), moderate performance for incomplete pattern recognition ($M = 0.58$, $SD = 0.43$), and comparatively low performance for growing structure recognition ($M = 0.16$, $SD = 0.35$). Correlations between the scales were small to moderate, suggesting related but distinct aspects of pattern recognition.

Inter-rater reliability was assessed by independently double coding a subset of responses, yielding substantial agreement (Cohen's $\kappa = .80$).

2.2.2 Mental rotation

Mental rotation ability was assessed using a standardized paper-pencil instrument—the MERRO instrument—introduced

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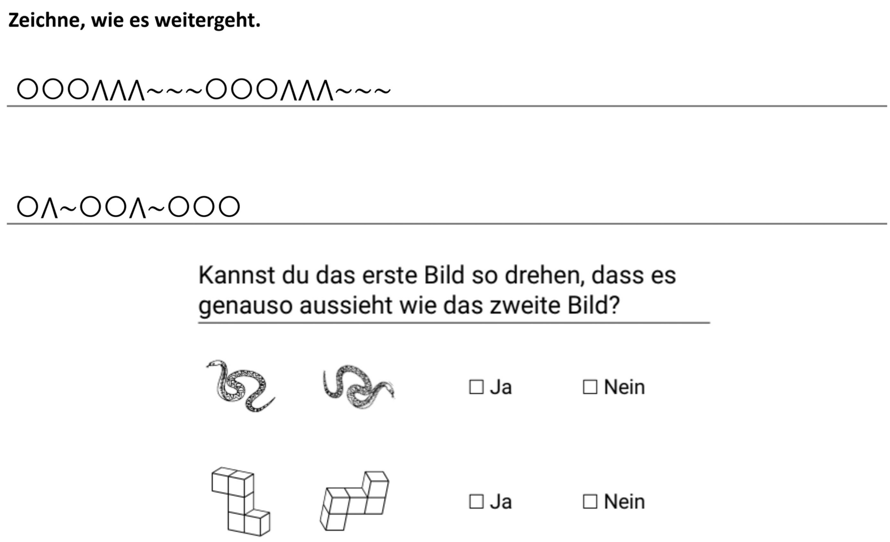

FIGURE 1
Sample items from the pattern recognition test (**a**, top) and the mental rotation test (**b**, bottom). Note. Top: Original German version of the item, stimulus translates as: "Draw how it continues". Bottom: Original German version of the item, stimulus translates as: "Can you rotate the first image so that it looks the same as the second image?"

in Reuter and Reinhold (2025; see the study for full details on item construction and validation). The instrument comprises 48 items designed to capture children's ability to mentally transform and match rotated objects under time constraints.

Each item presents a target figure alongside one comparison figure, which either corresponds to a rotated version of the target or to a rotated and mirrored image. Successful performance therefore requires mentally rotating the target representation and distinguishing it from mirror transformations.

The item set is based on a pool of figures that systematically vary across four contexts of increasing theoretical complexity. These include (0) authentic, object-like images, (1) cube-based figures with one or two bends, (2) cube-based figures with three planar bends, and (3) cube-based figures with three twisted bends. In addition, items vary in their mode of presentation: figures are shown either in a plain format or in perspective, with perspective items further differing in whether an additional tilt in the z-direction is introduced. This design allows for a systematic variation of spatial transformation demands, combining increasing structural complexity with more demanding spatial orientations. Figure 1b shows a sample item from the MERRO instrument as it was presented to the students.

The assessment was administered as a timed test (4 min). Students were instructed to decide whether the comparison figure matched the target (rotated) or not (mirrored). Responses were coded dichotomously at the item level (correct vs. incorrect), and scale scores were computed as the proportion of correctly solved items. In line with standard practice for timed assessments, items that were not reached or not answered were coded as incorrect.

The resulting scale demonstrated high internal consistency (Cronbach's $\alpha = .92$). Descriptive statistics indicated a mean solution rate of $M = 0.43$ ($SD = 0.21$). A one-sample t-test showed that performance was significantly below chance level (0.50), $t(386) = -6.45$, $p < .001$, 95% CI [0.41, 0.45], suggesting that the instrument successfully prevented random guessing and required genuine spatial processing.

2.3 Procedure

After the study was approved by the local school authority and the students as well as their parents gave informed consent to take part in the survey, the assessment was conducted during the regular mathematics lessons in the children's classrooms.

The pattern recognition test (i.e., MUFOR, Reuter and Reinhold, 2026) was administered as a group test in paper-based format with a 9-page test booklet in paper size A4. On the first page, the children indicated their grade level, their age in years and their gender. The following eight pages each contained five different items in randomized order; the order was identical for each participant. Children were asked to "draw how it continues" (in German: "Zeichne, wie es weitergeht."; see Reuter and Reinhold, 2026). To avoid influencing the children's pattern and structure recognition, no additional instructions were given. Each pattern sequence began on a simple writing line to provide a reference point without restricting their approach. Unlike boxes, which could have unintentionally suggested a fixed number of elements for continuation, the open writing line allowed for more flexibility—and accounted for age-related differences in fine motor skills. There was no time limit, and processing time was between about 15 and 30 min.

The mental rotation assessment (i.e., MERRO, Reuter and Reinhold, 2025) was administered subsequently as a group-based, paper-pencil test. The test booklet consisted of A4 pages with six items per page, each displaying a target figure alongside a comparison figure (rotated or mirrored) positioned directly next to it. Students were instructed to indicate whether the comparison figure matched the target or not. The assessment was timed, with a total duration of 4 min.

2.4 Analysis

To make the analytical research model explicit, the analysis distinguished between two related steps. In the profile-identification step, the three pattern-recognition measures—errorless individual pattern continuation, incomplete pattern recognition, and growing structure recognition—served as the variables on which students' developmental profiles were identified. In the subsequent covariate model, the resulting ordered profile membership was treated as the dependent variable. Grade level and mental rotation ability were included as independent variables reflecting developmental and cognitive predictors, whereas gender was included as an additional covariate.

All analyses were conducted in R. To investigate whether pattern recognition abilities form qualitatively distinct developmental profiles, we adopted a theory-driven, hypothesis-guided person-centered classification approach with exploratory cluster-based validation. This analytical strategy was chosen because the primary aim of the study was not to discover an unrestricted number of latent profiles, but to test whether children's performance corresponded to a theoretically derived three-level developmental model. Accordingly, the theory-driven classification served as the primary analytical step, whereas the data-driven cluster analysis was used as a robustness check to examine whether the same profile structure would also emerge from the empirical data.

A person-centered approach was chosen because the aim of the study was to identify qualitatively different profiles of pattern recognition abilities within students. Such approaches are particularly suitable when theoretical models assume that learners progress through distinct configurations of abilities rather than continuous variation along a single dimension. The analysis consisted of three steps: (1) theory-driven profile specification and classification, (2) data-driven cluster analysis for validation, and (3) covariate analysis predicting profile membership.

2.4.1 Theory-driven profile specification

Based on theoretical considerations regarding increasing complexity in pattern recognition, three developmental profiles were specified *a priori*. These profiles reflect hierarchical mastery of the three pattern recognition abilities measured in the MUFOR assessment: (i) errorless individual pattern continuation, representing the ability to produce a continuation that fulfills the general requirement of regularity, (ii) recognition of the basic unit in incomplete repeating patterns, and (iii) recognition of growing structures in pattern sequences. The three profiles represent increasingly advanced levels of pattern understanding and were operationalized as idealized response vectors across the three ability scales: $v(\text{i}) = (1,0,0)$, $v(\text{ii}) = (1,1,0)$ and $\text{v}(\text{iii}) = (1,1,1)$. To classify students according to the theoretical model, the Euclidean distance between each student's observed performance profile and each of the theoretical profiles was calculated. Each student was then assigned to the profile with the smallest distance, resulting in a theory-driven classification of students into developmental levels.

2.4.2 Data-driven cluster analysis

To examine whether the theoretically proposed profiles were supported by the empirical structure of the data, a *k*-means cluster analysis was conducted based on the same three ability variables (errorless pattern continuation, incomplete pattern recognition, growing structure recognition). While clustering, the number of clusters was explored using the {NbClust} package, which evaluates multiple cluster validity indices across a predefined range of possible cluster solutions. To assess the similarity between the theory-driven classification and the data-driven cluster solution, we calculated the Adjusted Rand Index (ARI) using the {flexclust} package. The ARI quantifies the agreement between two partitions of the same dataset by comparing all possible pairs of observations and evaluating whether they are assigned to the same cluster or to different clusters in both classifications. More specifically, the index counts the number of observation pairs that are grouped consistently across the two solutions (either together in both or separated in both) relative to those that are grouped inconsistently. The ARI ranges from −1 to 1, where values close to 1 indicate strong agreement between the classifications, values around 0 indicate agreement at the level expected by chance, and negative values indicate less agreement than expected by chance.

2.4.3 Descriptive profile characterization

To describe the identified profiles, the data-driven cluster analysis was used for further investigation. Descriptive statistics (means and standard deviations) were calculated for each of the three pattern recognition variables within the respective cluster and plotted using {ggplot2}.

2.4.4 Covariate analysis

To investigate predictors of developmental profile membership, we estimated an ordinal logistic regression model using the {MASS} package, using the ordered developmental clusters as the dependent variable, and predictor variables included grade level (treated as an ordered factor), gender (0 = male, 1 = female) and mental rotation ability (*z*-standardized metric score). Based on theoretical considerations, directed hypotheses were formulated for grade level and mental rotation ability, assuming that higher grade levels and stronger visuospatial abilities would be associated with higher levels of pattern recognition. For these predictors, confidence intervals were therefore interpreted in accordance with the directional expectations. In contrast, the effect of gender was examined exploratively, as no theoretically grounded directional hypothesis was formulated. Model coefficients were transformed into odds ratios, and corresponding confidence intervals were calculated.

3 Results

3.1 Individual stage hypothesis

Our first research question asked whether distinct individual profiles of pattern recognition abilities can be identified. More specifically, we examined whether children's individual performance across the three pattern recognition dimensions—errorless individual pattern continuation, incomplete pattern recognition, and growing structure recognition—can be meaningfully described by the theoretically proposed three-level stage model and whether this structure is also supported by the empirical data. The three profiles represent a theoretically derived hierarchical model of pattern recognition development, assuming that recognition of growing structures builds upon prior mastery of basic unit recognition, which itself presupposes the ability to produce regular pattern continuations.

In other words, the central test was whether children's responses formed the theoretically expected sequence of increasingly advanced pattern-recognition abilities: first producing regular continuations, then recognizing the basic unit in repeating patterns, and finally identifying growth structures.

We first addressed this question using the theory-driven profile assignment. Specifically, each student was assigned to the nearest of the three theoretically defined developmental profiles based on the squared deviation between the observed individual performance profile and the idealized stage profiles. Here, students' observed performance showed, on average, only small deviations from their assigned theoretical profile. Across all students, the squared deviation from the assigned theoretical profile was low on average, $M = 0.048$, $SD = 0.077$, with observed values ranging from zero to 0.565. Thus, the theory-driven classification suggests that the proposed developmental stage model captures students' individual performance profiles well.

We next examined whether the same general profile structure would also emerge from the data themselves. To do so, we explored the number of clusters supported by the empirical data across a range of possible cluster solutions. The answer was yes: the empirical data supported a three-profile solution. More specifically, 15 indices identified a three-cluster solution as optimal, whereas only two indices favored a two-cluster solution and no alternative cluster solution was supported by more than two indices. Thus, the majority of indices clearly supported a three-cluster structure, in line with the theoretically proposed model.

Finally, we examined how closely the data-driven cluster solution corresponded to the theory-driven classification. Indeed, both approaches converged almost perfectly. Agreement between the two classifications was high, $ARI = 0.954$. Because the empirical cluster analysis served as a validation of the hypothesis-guided classification rather than as a purely exploratory profile search, the interpretation focused on whether the cluster centroids reflected the theoretically expected ordering. This was the case: the first cluster was characterized by high performance in regular pattern continuation and low performance in the two structurally more demanding indicators; the second cluster showed high performance in regular continuation and basic-unit recognition, but low performance in

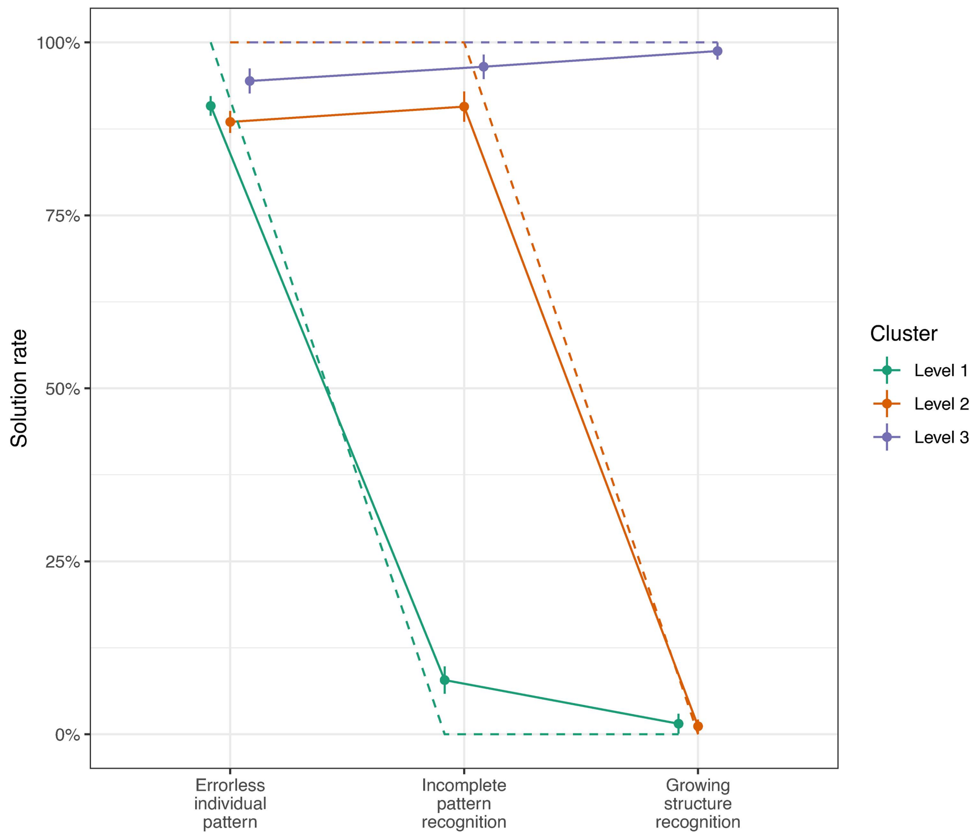

FIGURE 2
Empirical cluster profiles of pattern recognition abilities and theoretical stage vectors. Note. Points represent mean solution rates for each cluster across the three pattern recognition dimensions. Error bars indicate 95% confidence intervals. Solid lines depict empirical cluster profiles, whereas dashed lines represent the theoretically derived stage vectors for the three developmental levels.

TABLE 1 Cluster centroids.

Profile	Errorless continuation	Incomplete pattern recognition	Growing structure recognition
Level 1	0.91 (0.09)	0.08 (0.13)	0.02 (0.09)
Level 2	0.89 (0.11)	0.91 (0.15)	0.01 (0.06)
Level 3	0.94 (0.07)	0.96 (0.06)	0.99 (0.05)

growing-structure recognition; and the third cluster showed high performance across all three indicators. Cluster sizes were $n_1 = 157$, $n_2 = 176$, and $n_3 = 54$. Across the full sample of 387 students, only five students were assigned differently by the two approaches (all shifting from the Level 1 cluster to the Level 2 cluster). Thus, the findings were consistent across both analytical approaches: the theoretically expected three-level structure was not only imposed on the data but was also recovered empirically by the cluster analysis. Taken together, these findings provide strong support for the assumption that pattern recognition abilities can be described by three distinct and hierarchical developmental profiles at the individual level. Figure 2 shows the a-priori defined clusters and the empirically derived ones.

As shown in Table 1, the cluster-specific means closely matched the hypothesized hierarchical stage vectors. Level 1 was characterized by high performance only in errorless pattern continuation, Level 2 by high performance in errorless continuation and incomplete pattern recognition, and Level 3 by high performance across all three indicators.

3.2 Developmental and cognitive predictors hypothesis

Having established that the proposed stage structure is both theoretically plausible and empirically recoverable, the next question is whether membership in these profiles is systematically related to broader developmental and cognitive characteristics of the learners. More specifically, we asked whether students' membership in the identified developmental profiles is associated with grade level, mental rotation ability, and gender. To examine these relationships, we estimated an ordinal logistic regression model predicting profile membership from these predictors. The answer is largely consistent with our hypothesis. Overall, the model showed a meaningful fit, Nagelkerke $R^2 = .134$ (see Figure 3).

In substantive terms, this analysis examined whether children were more likely to reach higher pattern-recognition profiles when they were older, showed stronger visuospatial abilities, or differed by gender. As hypothesized, the linear component of the ordinal grade-level predictor (grades 1–4) was a significant predictor of profile membership: students in higher grades were more likely to belong to higher developmental profiles, $OR = 2.54$, 90% CI [1.75, 3.70]. Likewise, mental rotation ability was positively associated with higher profile membership, indicating that students with 1SD stronger visuospatial abilities then the sample average were 24% more likely to reach higher levels of pattern recognition, $OR = 1.24$, 90% CI [1.03, 1.49].

In addition, our explorative analysis of gender showed a significant association with profile membership, with girls being more likely than boys to belong to higher developmental profiles, $OR = 1.76$, 95% CI [1.19, 2.63].

4 Discussion

Pattern recognition is considered a key predictor of mathematical ability (Burgoyne et al., 2019; Fyfe et al., 2017; Kidd et al., 2014; Lüken, 2012; Rittle-Johnson et al., 2017; Wijns et al., 2021b; Zippert et al., 2019, 2020). For this reason, many studies have examined pattern recognition skills in children. Prior research has primarily focused on mean solution rates and item difficulty when examining age-related differences at the group level. However, it remains unclear whether children's pattern recognition abilities are organized into distinct configurations corresponding to individual profiles. Based on existing findings about the development and increasing complexity of pattern recognition, this study aims to determine if distinct individual profiles of pattern recognition can be identified.

4.1 Summary

Our findings confirm the hypothesized stage model, clearly demonstrating how pattern recognition can be identified in three

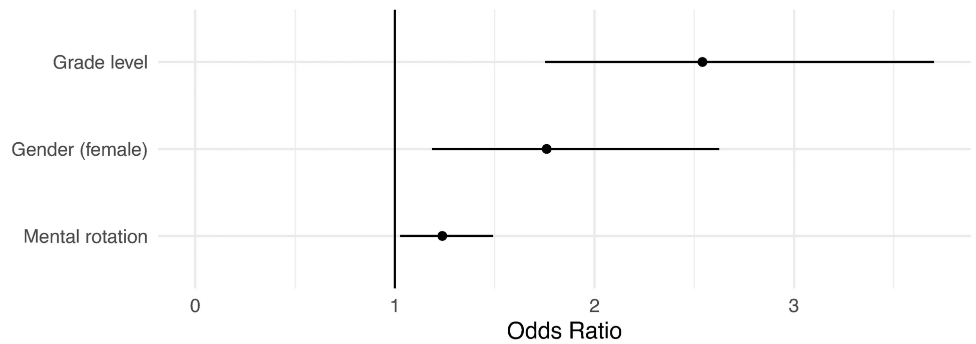

FIGURE 3
Odds ratios with confidence intervals for predictors of developmental profile membership (ordinal logistic regression).

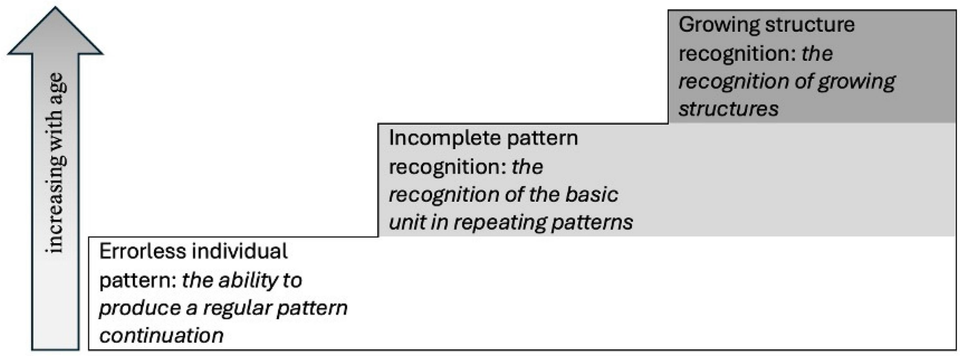

FIGURE 4
SHAPE model: stages of hierarchical advancement in pattern recognition, illustrating three levels of increasing complexity: errorless pattern continuation, incomplete pattern recognition, and growing structure recognition.

profiles. Three levels were confirmed based on these profiles using a combination of theory-testing and exploratory approaches (Figure 4).

In the initial stage, children have a concept of the regularity of patterns. This is evident in the fact that their continuations of pattern sequences are characterized by regular repetition, though it does not yet correspond to the basic unit. In the next stage, children can take the basic unit into account when continuing repeating pattern sequences. However, they cannot yet recognize structures in growing pattern sequences. In the final stage, children can identify structures in growing pattern sequences within continuations.

These findings confirm the results of previous studies that have shown the ability to recognize patterns continues to develop with age (Burgoyne et al., 2017; Clarke et al., 2008; Kyriakides and Gagatsis, 2003; Mulligan et al., 2020; Reuter and Reinhold, 2026). Furthermore, we were also able to demonstrate that higher mental rotation ability was positively associated with higher profile membership. This is supported by studies by Collins and Laskin (2015) and Rittel-Johnson and colleagues (2019), which have shown moderate correlations between visuospatial skills and pattern recognition skills, suggesting that visuospatial skills are used during pattern recognition. Our data also support the finding that recognizing growing structures is more difficult than recognizing repeating patterns (Pasnak et al., 2016; Reuter and Reinhold, 2026; Warren, 2005; Wijns et al., 2019, 2021b). At the same time, the profiles clearly demonstrate that children who cannot recognize the basic unit certainly cannot recognize growing structures.

The identified structural profiles of the SHAPE model provide empirical evidence for a hierarchical progression that aligns with and extends existing learning trajectories (Clements and Sarama, 2009). While traditional trajectories often focus on single activities like 'fixing' or 'extending', our three profilesdescribe qualitatively distinct configurations of structural understanding. The profiles correspond to Levels 3 and 4 of the 'Sounds of Intent' framework, which distinguish between the processing of single events and the recognition of meaningful groups or 'motifs' (McCarthy et al., 2025; Ockelford et al., 2025). The SHAPE model confirms this cross-domain logic due to the systematic variation in the structure of the pattern sequences used in the study. The model demonstrates that identifying a basic unit is a necessary structural prerequisite for more advanced pattern tasks.

Furthermore, our data also suggest a gender difference in pattern recognition. Specifically, girls in the age group considered in this study are more likely to be at a higher level. This finding is consistent with that of Lüken and Sauzet (2021), who also reported a significant gender difference in favor of girls in their study. One possible reason is that many other studies, such as those on mental rotation, involve time-constrained tasks. This time-factor was not present in our study, but it may trigger a competitive mindset that favors boys under certain circumstances, thereby affecting their performance (Cotton et al., 2013). Since many other studies on mathematics education find the opposite effect when reporting gender differences, it seems worthwhile to investigate this phenomenon further in future studies.

4.2 Limitations

While the three-stage SHAPE model is inherently conceptualized as describing longitudinal developmental trajectories, the data presented in this study are based on cross-sectional comparisons of students from different age groups rather than on true longitudinal observations of individuals. As such, the findings cannot provide direct empirical confirmation of individual developmental progressions across stages. At the same time, the observed patterns are consistent with the hypothesized developmental trajectories underlying the model. Future longitudinal research would be valuable to examine how closely individual development aligns with these trajectories, including the possibility of non-linear progressions such as skipping stages. Additionally, such work could explore how development may be supported through targeted intervention programs.

The covariates age and spatial ability suggest that older children and those with higher spatial ability are more likely to have a greater ability to recognize patterns and structures. However, given the available data, it is not possible to determine the extent to which these correlations exist with mathematical performance. A follow-up study assessing performance in addition to pattern recognition would be necessary to determine this.

The pattern sequences used in the study varied in various characteristics. For the repeating pattern sequences, the number of different symbols can be identified as the sole factor contributing to difficulty (Reuter and Reinhold, 2026). The pattern sequences used contained a maximum of three different symbols. It is conceivable that an increase in the number of symbols could lead to a shift in difficulty compared to the growing pattern sequences. This could have implications for the identified profiles and the associated level model.

This study focuses on pattern continuation as the only pattern-based activity. There are also a variety of other pattern-based activities, such as reproducing, extending, and extrapolating patterns. It would be interesting to see if the profiles can be applied to these activities as well.

Furthermore, pattern sequences involving numerical relationships present another set of challenges. For this reason, it is conceivable that the three-stage SHAPE model could be refined further, particularly with regard to various growing structures. Further research in this area is needed.

The three pattern-recognition indicators differed in their means and distributional properties. From the perspective of an unrestricted exploratory clustering approach, such differences could raise the concern that clusters are partly driven by measurement design. In the present study, however, the profiles were not derived from exploratory clustering alone but were specified *a priori* based on a hypothesis-guided developmental model. The observed distributional differences are therefore theoretically meaningful, because the model assumes increasingly demanding levels of structural understanding. Nevertheless, future studies should examine the robustness of the SHAPE profiles using additional indicators, longitudinal data, and alternative person-centered approaches.

4.3 Practical implementations

The identified sequence of levels enables us to place children at a specific level, thereby highlighting which patterns can be addressed as the next challenge. This evidence-based classification is helpful for designing learning situations because it provides a framework for structuring the learning process. The levels offer guidance on which pattern sequences to select for each child. Furthermore, when working with repeating pattern sequences at the second level, additional differentiation can be achieved based on the difficulty level, which is determined by the number of different symbols (Reuter and Reinhold, 2026). The added value of the SHAPE model lies in its integrated perspective on repeating and growing patterns. While earlier models often treated these as separate categories or introduced growing only as a late-stage numeric skill, our data-driven profiles show that they are part of a singular, hierarchical continuum of structural insight. Specifically, the SHAPE model highlights that children who have not yet mastered the identification of a basic unit are structurally unable to grasp growing sequences. This provides a clear pedagogical roadmap: rather than seeing growing patterns as a separate 'topic', they should be understood as a higher-order manifestation of regularity that requires the cognition of 'tertiary' relationships—the repeating of systematic change (Ockelford et al., 2025). By categorizing students into these structural profiles, practitioners can use a learning trajectories approach to provide targeted interventions that precisely match a child's current cognitive configuration. The equivalence between auditory and visuo-spatial processing suggests that children who struggle with pattern recognition in a mathematic contextmight benefit from musical pattern games that train the same underlying cognitive structures (McCarthy et al., 2025; Ockelford et al., 2025). Furthermore, evidence from the SoI framework indicates that a key hurdle in the trajectory—transitioning from mere regularity to a basic unit—is often a lack of awareness regarding the 'gap' or boundary between units. Explicitly focusing on these structural boundaries, a technique used in both music and math education, can facilitate the transition to more complex profiles.

Data availability statement

The raw data supporting the conclusions of this article will be made available by the authors, without undue reservation.

Ethics statement

The requirement of ethical approval was waived by the local school authority evaluated and approved the study (Regional Council of Freiburg, 7-6499.2). Detailed information about the study's aims and procedures was provided to the parents. All children gave verbal informed consent to take part in the study, after their parents gave written informed consent. Consent forms were signed before participation, ensuring that participants' rights and privacy were fully respected throughout the research process. for the studies involving humans because This research study did not require specific ethics committee approval according to national standards. These standards state that ethical approval is necessary if participants are exposed to risks, high physical or emotional stress, if they are not fully informed about the study's aims and procedures, if the study involves patients, the use of (f)MRI, electrical or magnetic stimulation, or psychopharmacological investigations. None of these conditions applied to our study, which involved minimal risk and fully informed participants, hence an ethics approval was not necessary. The studies were conducted in accordance with the local legislation and institutional requirements. Written informed consent for participation in this study was provided by the participants' legal guardians/next of kin.

Author contributions

DR: Conceptualization, Investigation, Methodology, Project administration, Resources, Validation, Visualization, Writing – original draft, Writing – review & editing. FR: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Project administration, Resources, Validation, Visualization, Writing – review & editing.

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The author(s) declared that this work was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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The author(s) declared that generative AI was not used in the creation of this manuscript.

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